

# 

### Semester Two Examination, 2018

### Question/Answer booklet

# MATHEMATICS

**SOLUTIONS**

**METHODS**

**UNITS 1 AND 2**

## Section Two:

## Calculator-assumed

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Student number: In figures |  |  |  |  |  |  |  |  |  |  |

In words

Your name

## Time allowed for this section

Reading time before commencing work: ten minutes

Working time: one hundred minutes

## Materials required/recommended for this section

***To be provided by the supervisor***

This Question/Answer booklet

Formula sheet (retained from Section One)

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,  
correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

## Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Section | Number of questions available | Number of questions to be answered | Working  time (minutes) | Marks available | Percentage of examination |
| Section One:  Calculator-free | 8 | 8 | 50 | 52 | 35 |
| Section Two:  Calculator-assumed | 13 | 13 | 100 | 98 | 65 |
|  | | |  | **Total** | 100 |

## Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Supplementary pages for the use of planning/continuing your answer to a question  
have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you do not use pencil, except in diagrams.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed 65% (98 Marks)

This section has**thirteen (****13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 (6 marks)

(a) A sequence is defined by . Determine

(i) the value of . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct value |

(ii) the sum of the first terms of this sequence. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ uses sum formula   correct sum |

(b) Another sequence is defined by Determine

(i) the value of . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct value |

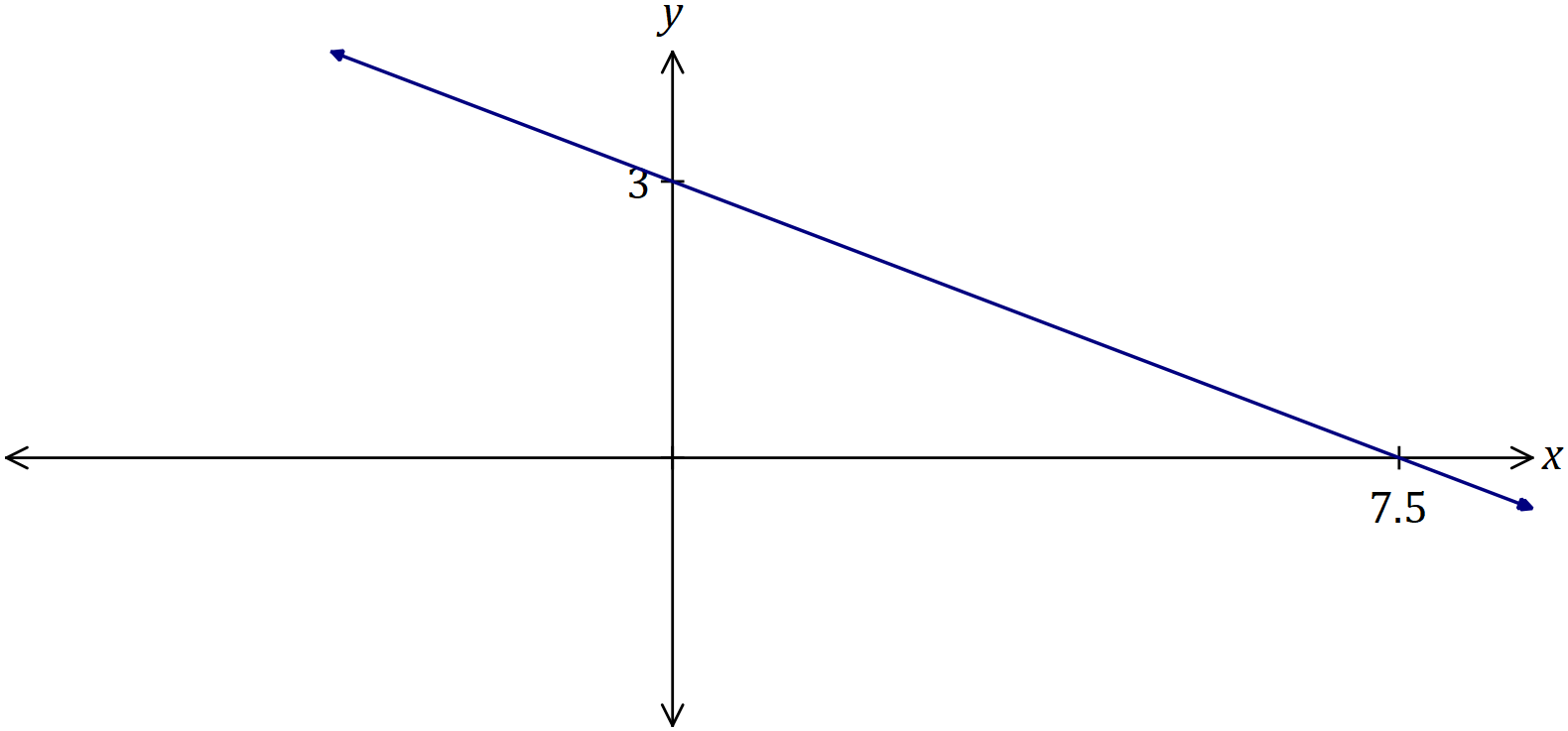
(ii) the value approaches as . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ uses sum to infinity formula   correct value |

Question 10 (6 marks)

The variables and are related by the equation .

(a) Sketch the graph of this relationship. (2 marks)



|  |
| --- |
| **Solution** |
| See graph |
| **Specific behaviours** |
| ✓ -intercept of line   -intercept of line |

(b) Express in terms of and briefly explain why is a function of . (2 marks)

|  |
| --- |
| **Solution** |
| For each value there is just one value, so one-to-one relationship. |
| **Specific behaviours** |
| ✓ correct rule   indicates use of vertical line test or 'one-to-one' relationship |

(c) The domain of is restricted to . State the range of . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct values   correct inequalities |

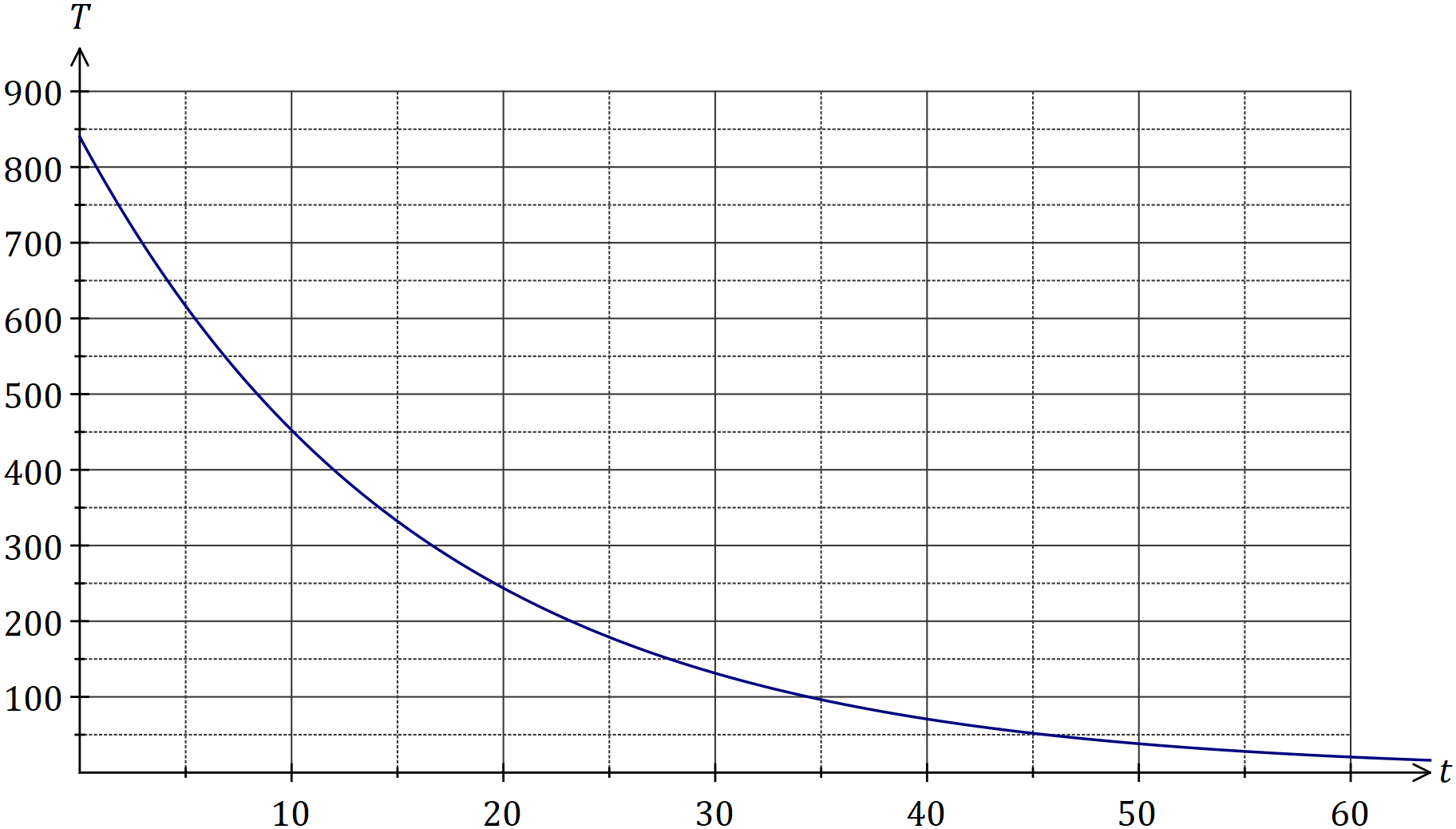
Question 11 (9 marks)

The temperature of a cast taken out of an oven cools according to the model , where is the time in minutes since the cast was removed from the oven. is measured in .

(a) Determine the fall in temperature of the cast during the first minutes. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
|  value of when  ✓ correct drop |

(b) Graph the temperature of the cast against time on the axes below. (4 marks)



|  |
| --- |
| **Solution** |
| See graph |
| **Specific behaviours** |
| ✓ -intercept   close to and   smooth curve throughout   no upward 'hook' at RHS |

(c) State the name of this type of function. (1 mark)

|  |
| --- |
| **Solution** |
| Exponential. |
| **Specific behaviours** |
| ✓ correct name |

(d) The temperature of the cast falls to room temperature of.

(i) Determine the time taken for the cast to reach room temperature. (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct time |

(ii) Comment on the usefulness of the model for large values of . (1 mark)

|  |
| --- |
| **Solution** |
| For large values of the model shows that but the temperature of the cast only falls to and so model not valid for large . |
| **Specific behaviours** |
| ✓ states not valid, with reason |

Question 12 (8 marks)

(a) Calculate the area of the minor segment that subtends an arc of in a circle of diameter cm. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ converts angle, uses correct radius   calculates area |

(b) A chord of length cm subtends an angle of at the centre of a circle. Calculate the radius of the circle. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ substitutes into formula   calculates radius |

(c) Parallelogram has side cm, side cm and an area of cm2. Determine the lengths of the diagonals of . (4 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ equation for half area   both angles of parallelogram   correct length of one diagonal   second correct length |

Question 13 (8 marks)

A mobile phone retailer classified recent sales of phones by the age of customer and if the phone was bought outright or on a plan. A summary of the data is shown in the table below.

|  |  |  |  |
| --- | --- | --- | --- |
|  | Aged under 30 | Aged 30 or over | Total |
| Bought outright |  |  |  |
| Bought on a plan |  |  |  |
| Total |  |  |  |

(a) Determine the values of and shown in the table. (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ each correct value |

(b) A recent sale is selected at random from those recorded above. Event occurs if the customer was aged under 30 and event occurs if the phone was bought outright. Determine the following probabilities:

(i) . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct probability |

(ii) . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct probability |

(iii) . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct probability |

(iv) . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ numerator   denominator |

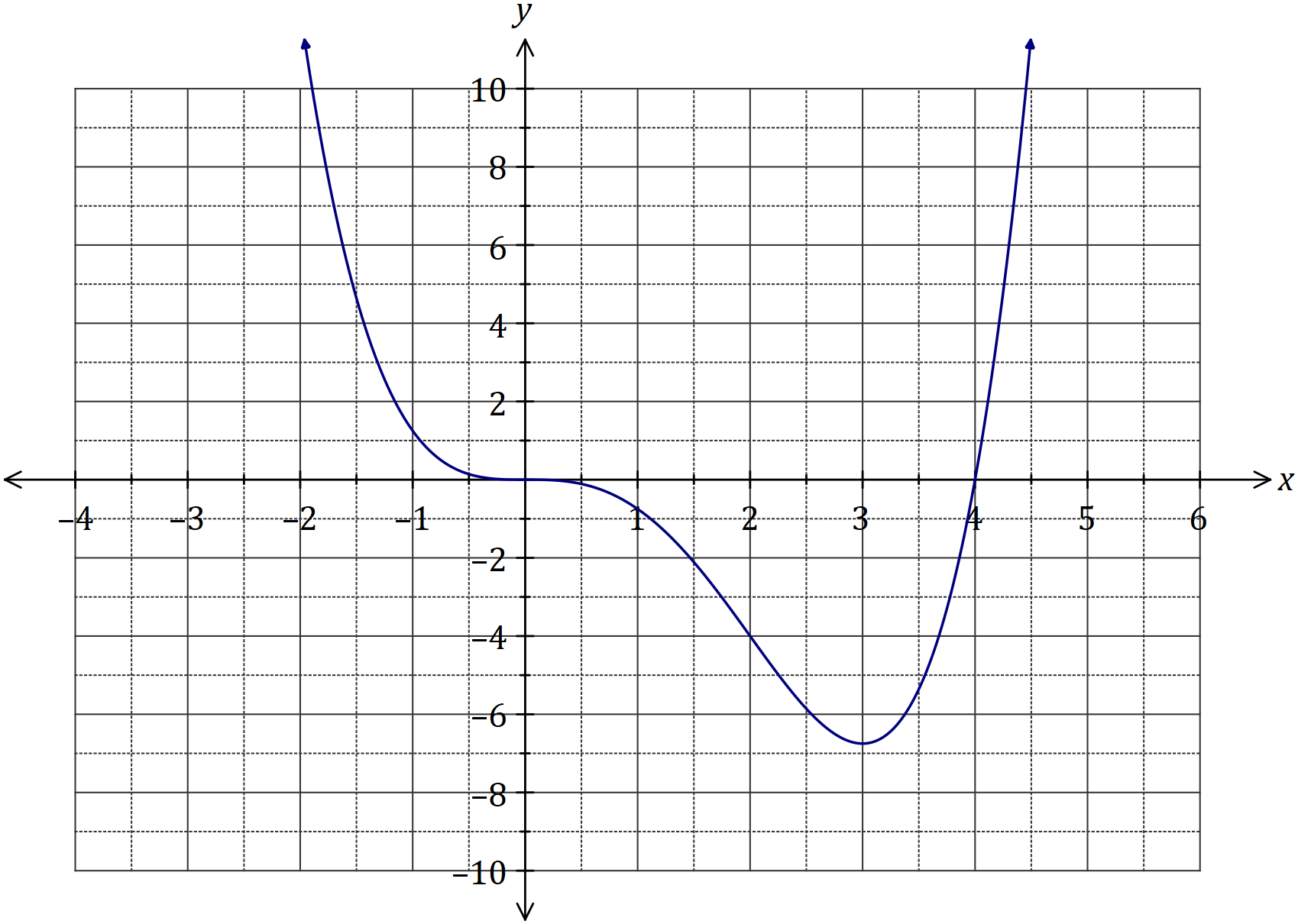
Question 14 (7 marks)

A function is defined by .

(a) Use the derivative to determine the coordinates of all stationary points of the function. (3 marks)

|  |
| --- |
| **Solution** |
| Stationary points at and |
| **Specific behaviours** |
| ✓ correct derivative   correct zeros of derivative   correct coordinates |

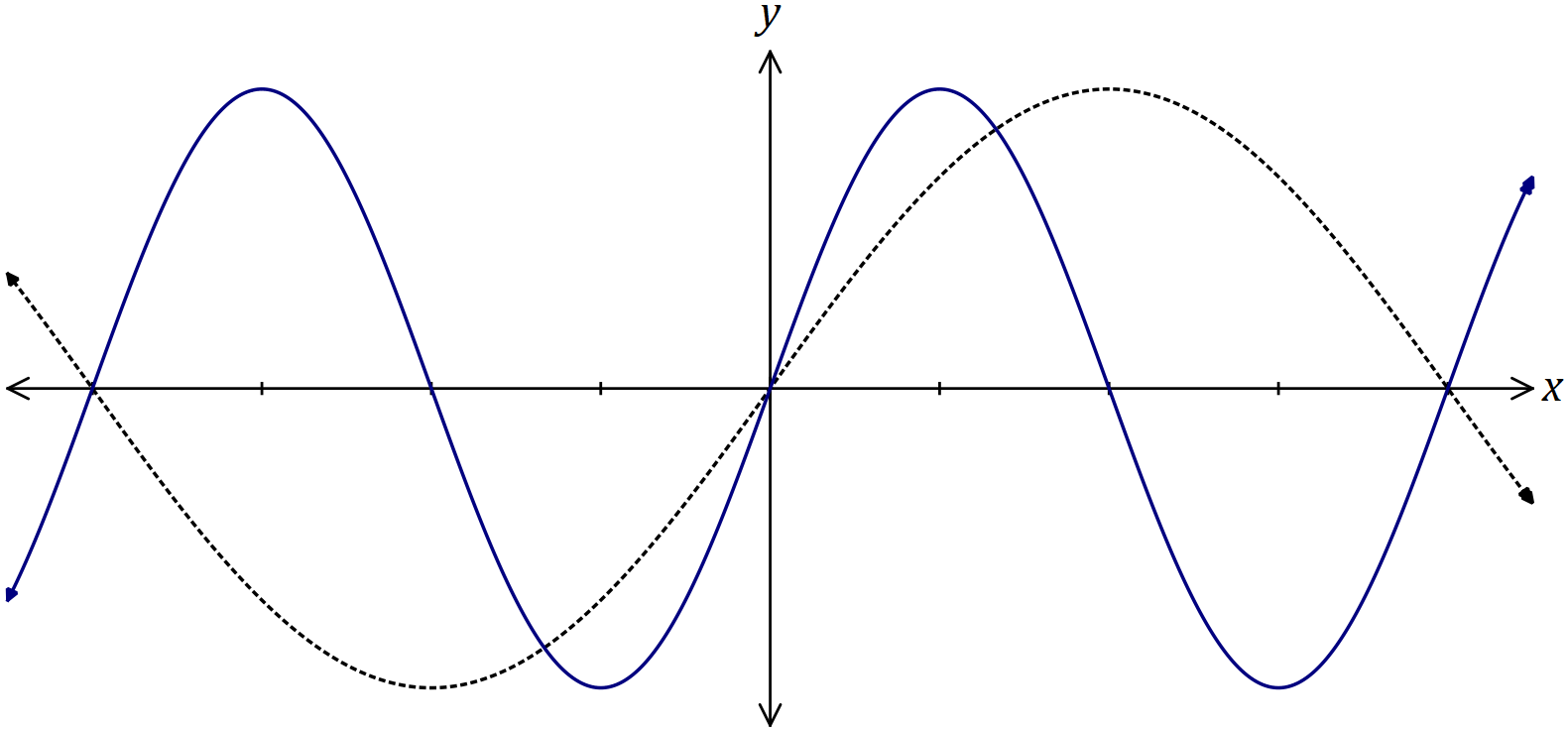
(b) Sketch the graph of on the axes below. (4 marks)



|  |
| --- |
| **Solution** |
| See graph |
| **Specific behaviours** |
| ✓ HPI at (0, 0)   minimum at   root at   correct curvature |

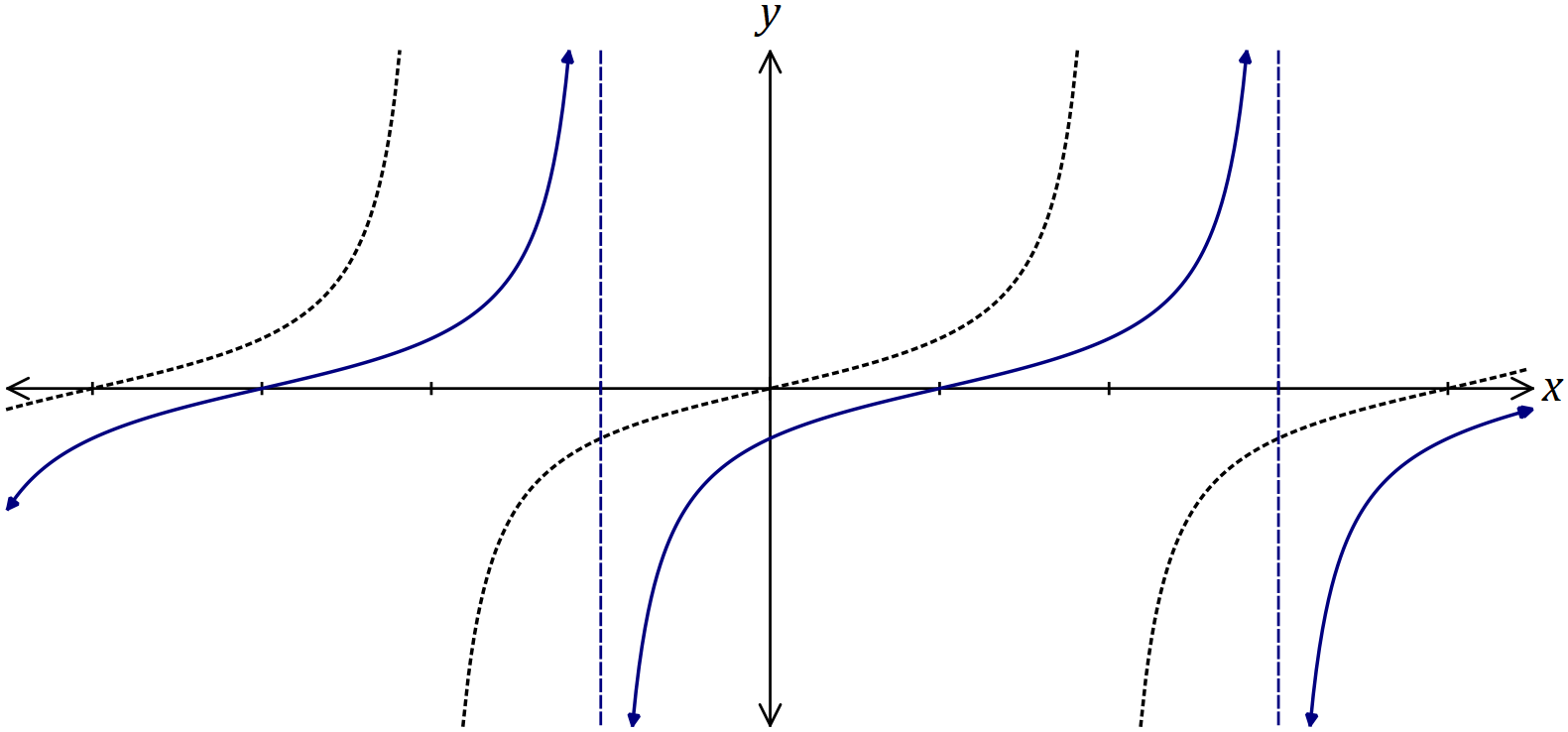
Question 15 (7 marks)

(a) The graph of is shown below. On the same axes, sketch . (2 marks)



|  |
| --- |
| **Solution** |
| See graph |
| **Specific behaviours** |
| ✓ location of roots   correct amplitude/shape |

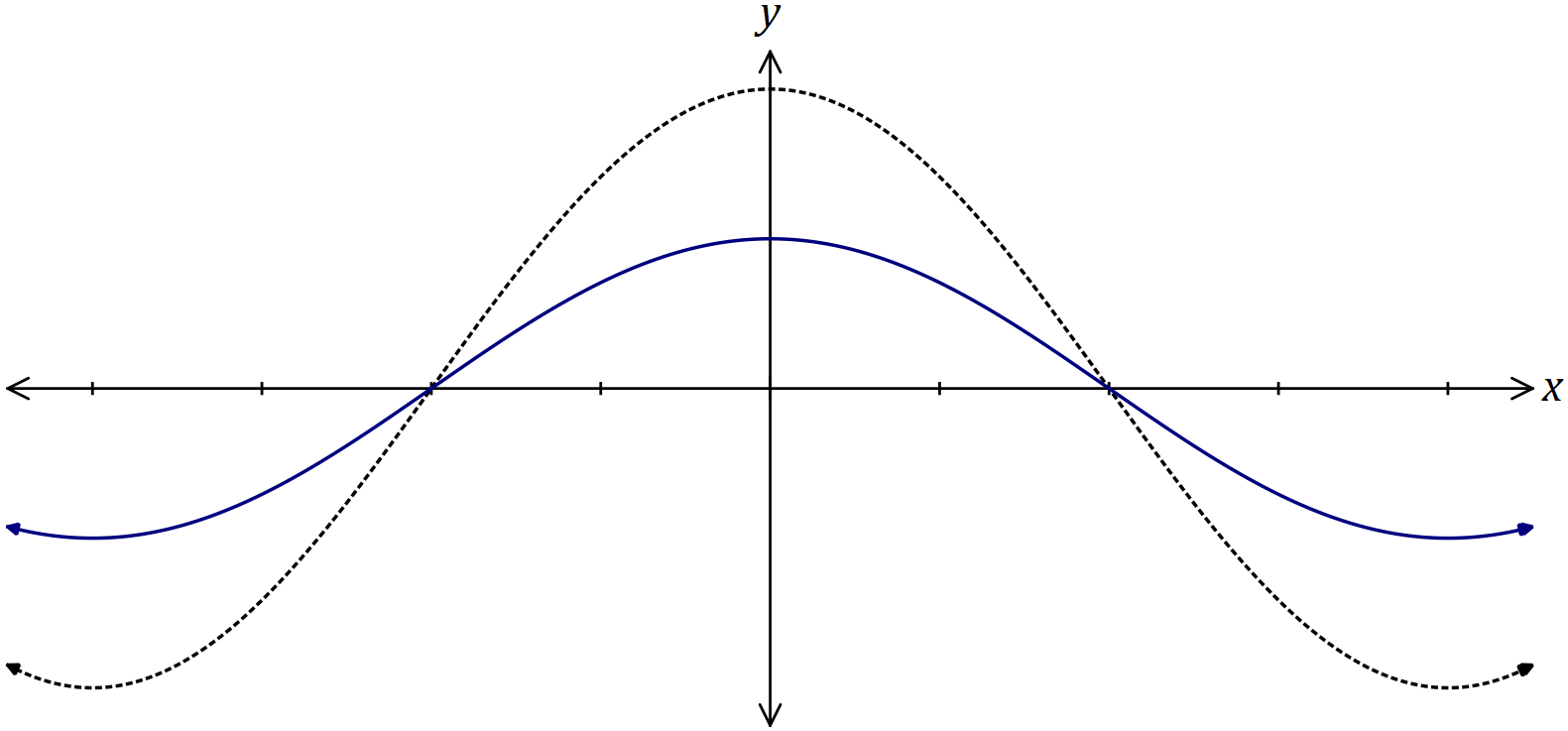
(b) The graph of is shown below. On the same axes, sketch , and all its asymptotes. (3 marks)



|  |
| --- |
| **Solution** |
| See graph |
| **Specific behaviours** |
| ✓ translation   location of roots   asymptotes |

(c) The graph of is shown below. On the same axes, sketch .

(2 marks)



|  |
| --- |
| **Solution** |
| See graph |
| **Specific behaviours** |
| ✓ location of roots   correct amplitude/shape |

Question 16 (8 marks)

A council took a random sample of and properties from suburbs and respectively. A total of of the properties in the sample were in arrears with their rates, and of these properties were in suburb . 'In arrears' means that payment of rates is overdue.

(a) Council officers wanted to choose of the properties that were in arrears. How many different selections of properties are possible? (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ indicates use of formula   correct number |

(b) Determine the probability that one randomly chosen property from the sample

(i) is in suburb and is in arrears. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ numerator   denominator |

(ii) is not in arrears given that it is in suburb . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct probability |

(c) Justifying your answer with conditional probabilities, comment on whether being in arrears with rates is independent of the suburb the property is in. (3 marks)

|  |
| --- |
| **Solution** |
| Hence being in arrears is independent of suburb, as conditional probabilities are very similar. |
| **Specific behaviours** |
|  calculates  ✓ calculates   correct conclusion |

Question 17 (8 marks)

Two water containers, initially empty, are being filled with water. The amount of water added to container each minute follows an arithmetic sequence, with mL poured in during the first minute and mL poured in during the second minute. The amount of water added to container each minute follows a geometric sequence, with mL poured in during the first minute and mL poured in during the second minute.

(a) The amount of water poured into container during the minute is given by . State the value of the constants and . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ value of   value of |

(b) Determine the total amount of water in container at the end of the minute.

(2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ uses sum formula   correct amount |

(c) How long does it take to fill container with mL of water? (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ uses sum formula   correct time |

(d) Container first holds more water than container at the end of minute .

(i) Determine the value of . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct value |

(ii) State, to the nearest mL, how much more water contains than at this time.

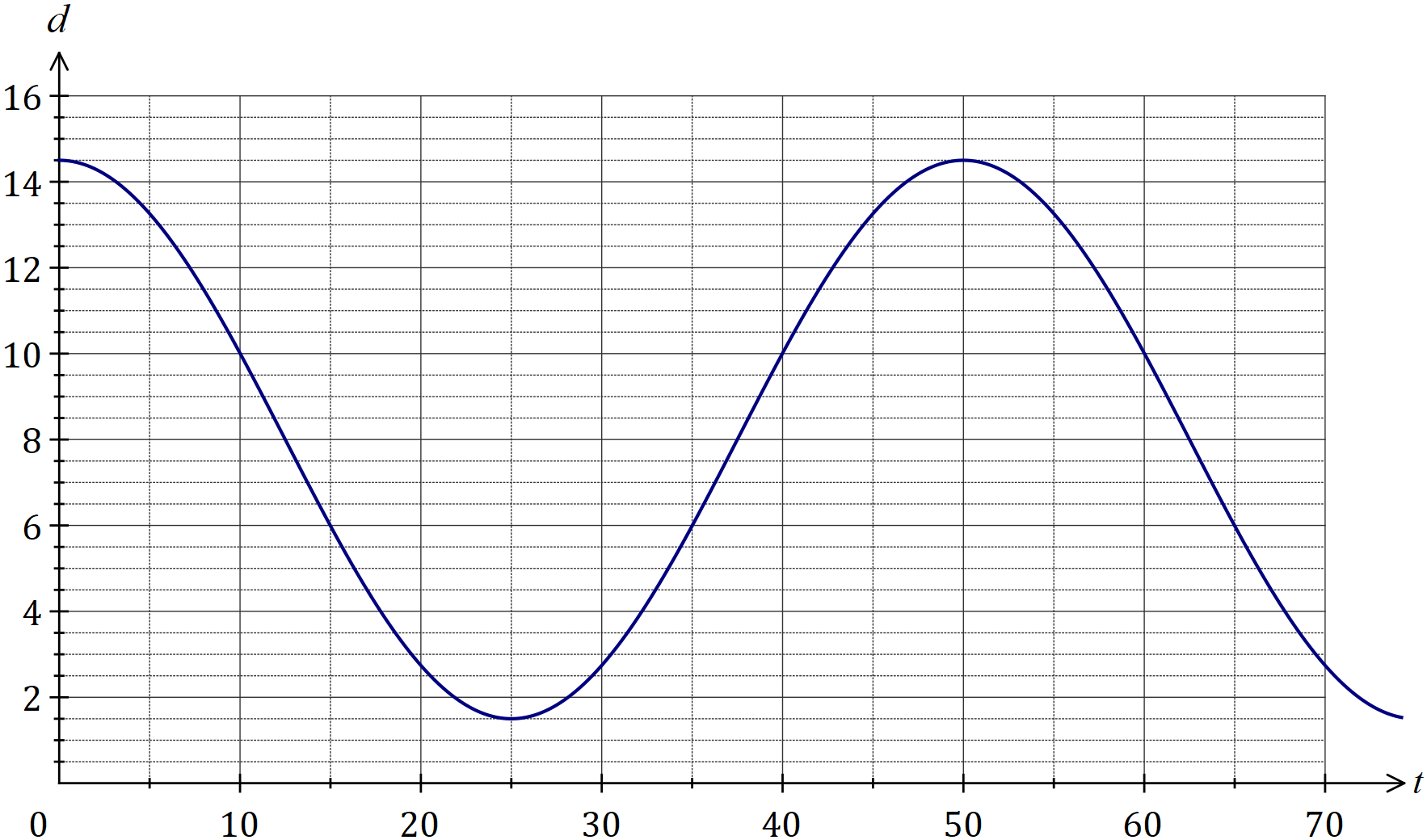
(1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct value |

Question 18 (8 marks)

The height, metres, above level ground of a seat on a steadily rotating Ferris wheel seconds after observations began was given by

(a) Draw the graph of the height of the seat against time on the axes below. (4 marks)



|  |
| --- |
| **Solution** |
| See graph |
| **Specific behaviours** |
|  correct -intercept  ✓ accurate curve near maximum  ✓ accurate curve near minimum  ✓ smooth curve throughout |

(b) How long did the Ferris wheel take to complete one revolution? (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct time |

(c) At what time, when the seat was rising, did it first reach a height of metres? (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ time that rounds to |

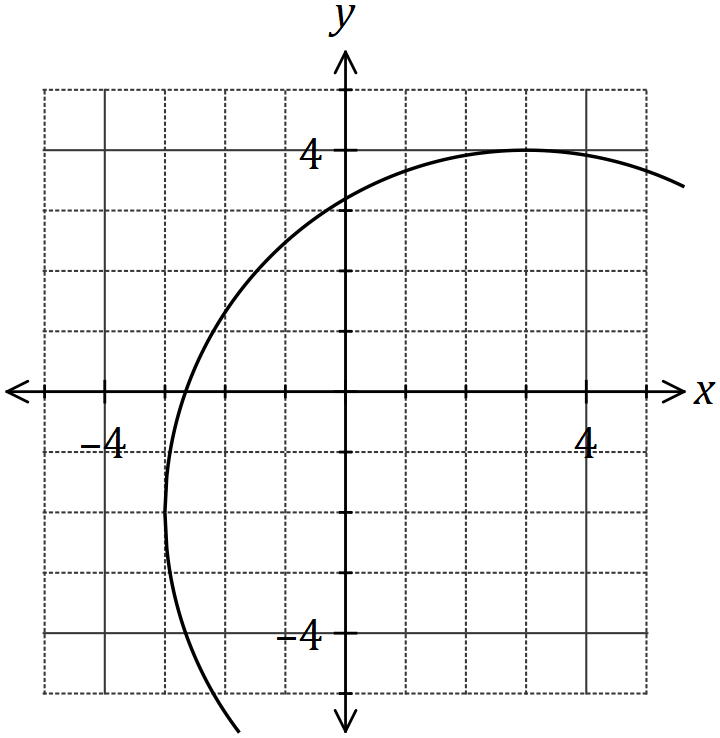
(d) Determine the change in height of the seat between and , giving your answer rounded to the nearest cm. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ determines both heights  ✓ states difference to nearest cm |

Question 19 (7 marks)

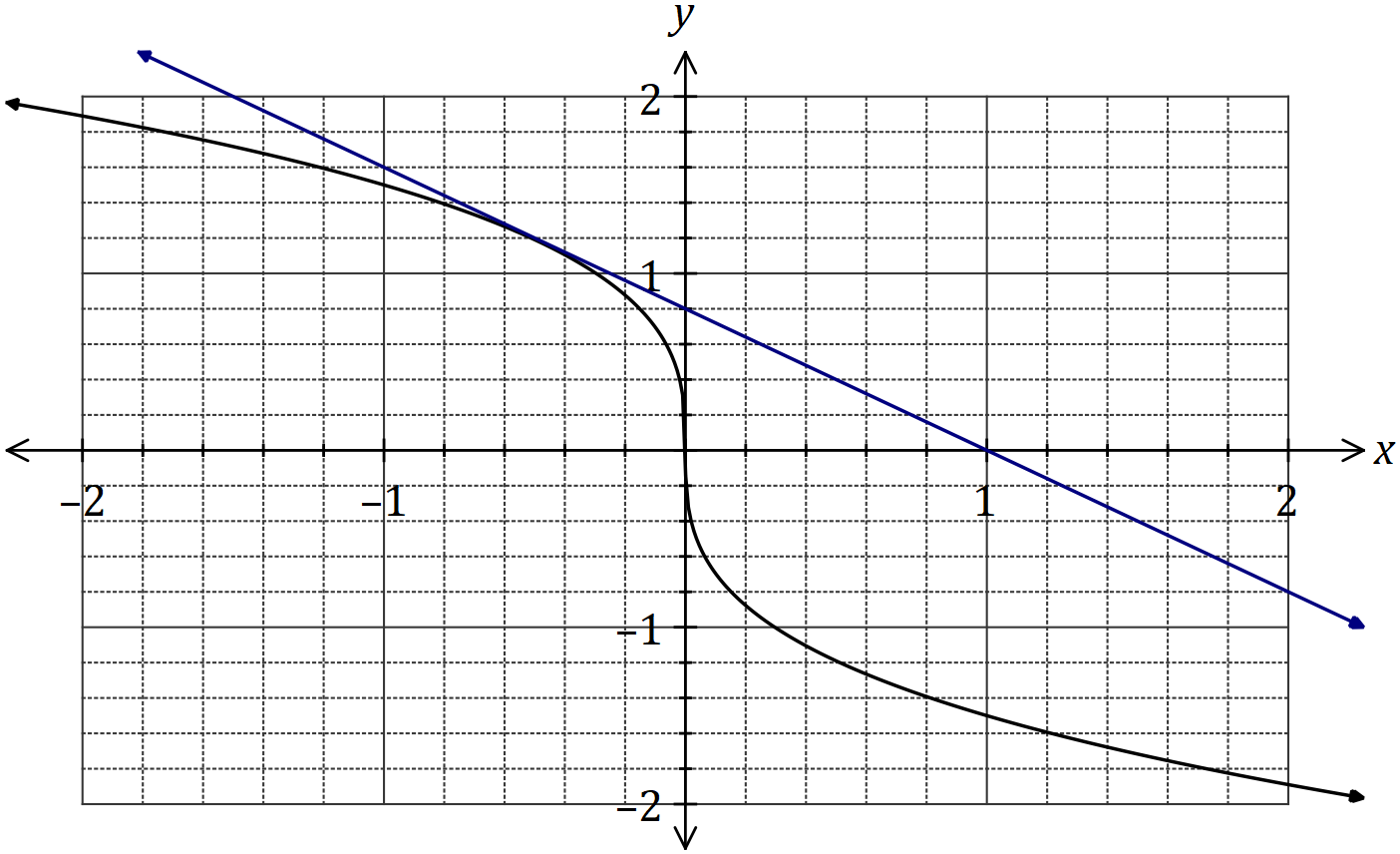
(a) Part of the circle is shown below. Determine the values of the constants and . (4 marks)

|  |
| --- |
| **Solution** |
| See graph |
| **Specific behaviours** |
| ✓ tangent drawn with ruler |



|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ circle in factored form   correct radius and centre   expands into required form   correct values of and |

(b) The graph of a power function is shown below.



(i) Draw the tangent to the curve when . (1 mark)

(ii) Use the tangent to estimate . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ uses gradient of tangent   value |

Question 20 (8 marks)

A diagnostic test for a disease has a chance of giving the correct outcome and it is known that of all sheep on a station have the disease. It can be assumed that the correct outcome of the test is independent of whether a sheep has the disease.

(a) A sheep is randomly selected for the test from those on the station. Determine the probability that

(i) the sheep has the disease, but the test indicates that it does not. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ indicates multiplication of correct probabilities   correct probability |

(ii) the sheep actually has the disease if the test indicates that it does. (4 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ calculates  ✓ calculates   indicates   correct probability |

(b) Two sheep are randomly selected for the test from those on the station. Determine the probability that just one of the sheep is diagnosed correctly. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ indicates correct method   correct probability |

Question 21 (8 marks)

A pyramid with a rectangular base of length and width has perpendicular height . The length of the base is three times its width and the sum of the width, length and height is cm.

(a) Calculate the length, height and volume of the pyramid when cm. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct length and height   correct volume |

(b) Show that the volume of the pyramid is given by . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ expressions for length and height   substitutes width, length and height correctly |

(c) Use calculus to determine the maximum volume of the pyramid and state the dimensions required to achieve this volume. (4 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct derivative using given variables   solves derivative equal to zero   correct maximum volume   correct dimensions |

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_

Supplementary page

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